01. (a) $P(x) = x^2 - 2\lambda x + 2\lambda - 1$

[illegible] $\Delta = 4\lambda^2 - 4.1(2\lambda - 1)$

$= 4(\lambda^2 - 2\lambda + 1)$

$= 4(\lambda - 1)^2 \geq 0;\ \forall \lambda \in \mathbb{R}$

$\therefore$ $p(x) = 0$ [illegible]

$P(x) = 0$ [illegible] α [illegible] β [illegible]

[illegible] $\alpha + \beta = 2\lambda$ [illegible] $\alpha\beta = 2\lambda - 1$

$\alpha + \beta = \alpha^2 + \beta^2$ [illegible]

$\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta$

[illegible] $2\lambda = 4\lambda^2 - 2(2\lambda - 1)$

$\Rightarrow 4\lambda^2 - 6\lambda + 2 = 0$

$\Rightarrow 2\lambda^2 - 3\lambda + 1 = 0$

$\Rightarrow (2\lambda - 1)(\lambda - 1) = 0$

$\therefore \lambda = \frac{1}{2}$ [illegible] $\lambda = 1$

λ [illegible] $\lambda = \frac{1}{2}$ [illegible] $\lambda = 1$ [illegible]

(b) $P(x)$, $(x - 1)$, $(x - 2)$ [illegible] $(x - 3)$ [illegible] $1, \frac{1}{2}$ [illegible] $\frac{1}{3}$ [illegible]

$P(1) = 1,\ P(2) = \frac{1}{2}$ [illegible] $P(3) = \frac{1}{3}$

$Q(x) = x P(x) - 1$ [illegible]

$Q(1) = 1.P(1) - 1$

$= 1.1 - 1 = 0$

$\therefore (x - 1)$, $Q(x)$ [illegible]

$Q(2) = 2.P(2) - 1$

$= 2.\frac{1}{2} - 1 = 0$

$\Rightarrow (x - 2)$, $Q(x)$ [illegible]

$Q(3) = 3.\frac{1}{3} - 1 = 0$

$\Rightarrow (x - 3)$ [illegible] $Q(x)$ [illegible]

$Q(x) = x P(x) - 1$ [illegible] $P(x)$ [illegible] $Q(x)$ [illegible]

[illegible] $Q(x) = k(x - 1)(x - 2)(x - 3)$ [illegible]

[illegible] $k(x - 1)(x - 2)(x - 3) = x P(x) - 1$

$x = 0$ [illegible] $k(-1)(-2)(-3) = -1$

$k = \frac{1}{6}$

$\therefore Q(x) = \frac{1}{6}(x - 1)(x - 2)(x - 3)$

02. (a) [illegible]

$= {}^9C_6 = \frac{9!}{3!6!}$

$= \frac{9 \times 8 \times 7 \times 6!}{3 \times 2 \times 1 \times 6!}$

$= 84$

(i) [illegible]

$= {}^6C_3 = \frac{6!}{3!3!}$

$= \frac{6 \times 5 \times 4 \times \cancel{3!}}{3 \times 2 \times 1 \times \cancel{3!}} = 20$

(ii) [illegible] 5 [illegible]

- [illegible] 5 [illegible] 4 [illegible]
- [illegible] 5 [illegible] 4 [illegible]

[illegible]

$= 1 \times {}^4C_1 + {}^5C_4 \times {}^4C_2$

$= 1 \times 4 + \frac{5!}{4!1!} \times \frac{4!}{2!2!}$

$= 4 + \frac{5 \times \cancel{4}^{2} \times 3 \times \cancel{2} \times 1}{\cancel{2} \times 1 \times \cancel{2} \times 1}$

$= 4 + 30 = 34$

(b) $(1 + 7x)^{23} = A_0 + A_1 x + \dots + A_r x^r + \dots + A_{23} x^{23}$

[illegible] $A_r = {}^{23}C_r . 7^r$

(i) $\frac{A_{r+1}}{A_r} = \frac{{}^{23}C_{r+1}\ 7^{r+1}}{{}^{23}C_r\ 7^r}$

$= 7 \frac{23!}{(22 - r)!(r + 1)!} . \frac{(23 - r)!\,r!}{23!}$

$= \frac{7(23 - r)}{(r + 1)}$

$= \frac{161 - 7r}{r + 1}$

$\frac{A_{r+1}}{A_r} \gtreqless 1 \Leftrightarrow \frac{161 - 7r}{r + 1} \gtreqless 1$

$\Leftrightarrow 161 - 7r \gtreqless r + 1$

$\Leftrightarrow 160 \gtreqless 8r$

$\Leftrightarrow 20 \gtreqless r$

[illegible] $r < 20 \Rightarrow A_{r+1} > A_r$

$r = 20 \Rightarrow A_{r+1} = A_r$

$r > 20 \Rightarrow A_{r+1} < A_r$

$\therefore$ [illegible] $A_{21} = A_{20}$

$\therefore$ [illegible]

$A_{20} = A_{21} = {}^{23}C_{20} . 7^{20}$

$= 1771 \times 7^{20}$

[illegible] 21 [illegible] 22 [illegible]

(ii) $(1 + 7x)^{23}$ [illegible] $r + 1$ [illegible] T_{r+1}

[illegible] $T_{r+1} = {}^{23}C_r (7x)^r = {}^{23}C_r 7^r x^r$

[illegible] T_4 [illegible]

$T_3 < T_4 > T_5$

$\Rightarrow {}^{23}C_2\, 7^2 x^2 < {}^{23}C_3\, 7^3 x^3$

[illegible] ${}^{23}C_3\, 7^3 x^3 > {}^{23}C_4\, 7^4 x^4$

$\Rightarrow \frac{23!}{21!\,2!} < \frac{23!}{20!\,3!} 7x$ [illegible]

$\frac{23!}{19!\,4!} 7x < \frac{23!}{20!\,3!}$

$\Rightarrow \frac{\cancel{23!}}{\cancel{21}\ \cancel{20!}\,\cancel{2!}} < \frac{\cancel{23!}}{\cancel{20!}\,\cancel{3}\,\cancel{2!}} 7x$ [illegible]

$\frac{\cancel{23!}}{\cancel{19!}\,\cancel{4}\,\cancel{3!}} \times 7x < \frac{\cancel{23!}}{\cancel{20}\ \cancel{19!}\,\cancel{3!}}$

$\Rightarrow \frac{1}{49} < x$ [illegible] $x < \frac{1}{35}$

$\Rightarrow \frac{1}{49} < x < \frac{1}{35}$

03. (a) $n = 1$ [illegible]

L. H. S $= \frac{1}{3}$, R. H. S $= \frac{3}{4} - \frac{1}{4} - \frac{1}{6}$

$= \frac{18 - 6 - 4}{24} = \frac{8}{24} = \frac{1}{3}$

[illegible] $n = 1$ [illegible]

$n = P$ [illegible]

[illegible] P [illegible]

[illegible] $\frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(P + 1)^2 - 1}$

$= \frac{3}{4} - \frac{1}{2(P + 1)} - \frac{1}{2(P + 2)}$

[illegible] $\frac{1}{(P + 2)^2 - 1}$ [illegible]

$\frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \dots + \frac{1}{(P + 1)^2 - 1} + \frac{1}{(P + 1 + 1)^2 - 1}$

$= \frac{3}{4} - \frac{1}{2(P + 1)} - \frac{1}{2(P + 2)} + \frac{1}{(P + 2)^2 - 1}$

$= \frac{3}{4} - \frac{1}{2(P + 1)} - \frac{1}{2(P + 2)} + \frac{1}{(P + 1)(P + 3)}$

$= \frac{3}{4} - \cancel{\frac{1}{2(P + 1)}} - \frac{1}{2(P + 2)} + \cancel{\frac{1}{2(P + 1)}} - \frac{1}{2(P + 3)}$

$= \frac{3}{4} - \frac{1}{2(P + 1 + 1)} - \frac{1}{2(P + 1 + 2)}$

$\therefore n = P$ [illegible] $n = P + 1$ [illegible]

$\therefore$ [illegible] n [illegible]

$\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{r(r + 2)}$

$= \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{(r + 1)^2 - 1}$

$= \lim_{n \to \infty} \left(\frac{3}{4} - \frac{1}{2(n + 1)} - \frac{1}{2(n + 2)}\right)$

$= \frac{3}{4}$

$\sum_{r=1}^{\infty} \frac{1}{r(r + 2)}$ [illegible]

[illegible] $= \frac{3}{4}$

(b) $|x - 1| = \begin{cases} x - 1 & x \geq 1 \\ -(x - 1) & x < 1 \end{cases}$

$\left|\frac{1}{2}x + 1\right| = \begin{cases} \frac{1}{2}x + 1 & x \geq -2 \\ -\left(\frac{1}{2}x + 1\right) & x < -2 \end{cases}$

(i) $x < -2$ [illegible]

[illegible] $-(x - 1) + \frac{x}{2} + 1 < 1$ [illegible]

[illegible] $x > 2$ [illegible]

(ii) $-2 \leq x < 1$ [illegible]

[illegible]

$-(x - 1) - \left(\frac{x}{2} + 1\right) < 1$ [illegible]

[illegible] $x > -\frac{2}{3}$

[illegible] $x \in \left(-\frac{2}{3}, 1\right)$ [illegible]

(iii) $x \geq 1$ [illegible]

[illegible] $(x - 1) - \left(\frac{x}{2} + 1\right) < 1$ [illegible]

[illegible] $x < 6$

[illegible] $x \in [1, 6)$ [illegible]

$\therefore$ [illegible] x [illegible]

$\left\{x : -\frac{2}{3} < x < 6\right\} = \left(-\frac{2}{3}, 6\right) \rightarrow$

[illegible] 5 [illegible]

04. $2z^2 = \frac{1}{2}(1 + \sqrt{3}i)^2 = -1 + \sqrt{3}i$

$= 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

[illegible] $2z^2$ [illegible] 2 [illegible]

$\frac{2\pi}{3}$ [illegible]

$\frac{3}{z^2} = \frac{3 \times 2}{(-1 + \sqrt{3}i)} = \frac{6}{(-1 + \sqrt{3}i)} \times \frac{(-1 - \sqrt{3}i)}{(-1 - \sqrt{3}i)}$

$= \frac{6}{(1+3)}(-1-\sqrt{3}\,i)$

$= \frac{3}{2}(-1-\sqrt{3}\,i) \quad = 3\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\,i\right)$

$= 3\left[\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right]$

[illegible] $\frac{3}{z^2}$ [illegible] 3 [illegible] $-\frac{2\pi}{3}$

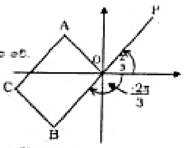
P [illegible] z [illegible]

$P\hat{O}B = \left|\text{Arg } Z - \text{Arg}\left(\frac{3}{z^2}\right)\right|$

$= \left|\frac{\pi}{3} - \left(-\frac{2\pi}{3}\right)\right|$

$= \pi$

$\therefore$ P, O, B [illegible]



C [illegible]

$= 2z^2 + \frac{3}{z^2}$

$= -1 + \sqrt{3}\,i - \frac{3}{2}(1+\sqrt{3}\,i)$

$= -\frac{5}{2} - \frac{\sqrt{3}\,i}{2}$

CO [illegible] $= \sqrt{\frac{25}{4} + \frac{3}{4}} = \frac{\sqrt{28}}{2} = \sqrt{7}$

AB [illegible] $= \left|2z^2 - \frac{3}{z^2}\right|$

$= \left|-1 + \sqrt{3}\,i + \frac{3}{2} + \frac{3\sqrt{3}}{2}\,i\right|$

$= \left|\frac{1}{2} + \frac{5\sqrt{3}}{2}\,i\right|$

$= \sqrt{\frac{1}{4} + \frac{75}{4}} = \sqrt{19}$

05. (a) $y = e^{-x}(\cos 2x + \sin 2x)$

$\frac{dy}{dx} = e^{-x}(-2\sin 2x + 2\cos 2x) + (\cos 2x + \sin 2x)(-e^{-x})$

$= e^{-x}(-3\sin 2x + \cos 2x)$

$\frac{dy}{dx} + y = -3e^{-x}\sin 2x + e^{-x}\cos 2x + e^{-x}\cos 2x + e^{-x}\sin 2x$

$= 2e^{-x}\cos 2x - 2e^{-x}\sin 2x$

$= 2e^{-x}(\cos 2x - \sin 2x)$

$\therefore \frac{d^2y}{dx^2} + \frac{dy}{dx} = 2e^{-x}(-2\sin 2x - 2\cos 2x) + 2(\cos 2x - \sin 2x)(-e^{-x})$

$= -4e^{-x}(\sin 2x + \cos 2x) - 2e^{-x}(\cos 2x - \sin 2x)$

$= -4y - \left(\frac{dy}{dx} + y\right)$

$\therefore \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0 \qquad p = 2 \quad q = 2$

$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

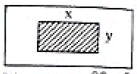
$y(0) = 1, \quad \left(\frac{dy}{dx}\right)_{x=0} = 1.$

$\left(\frac{d^2y}{dx^2}\right)_{x=0} = -2\left(\frac{dy}{dx}\right)_{x=0} - 5y(0) = -7$

$\therefore \left(\frac{d^3y}{dx^3}\right)_{x=0} = -2\left(\frac{d^2y}{dx^2}\right)_{x=0} - 5\left(\frac{dy}{dx}\right)_{x=0}$

$= +14 - 5 \qquad = 9$

(b) [illegible] x cm [illegible] y cm [illegible]



එවිට [illegible] $(x+12)$ cm [illegible] $(y+16)$ cm [illegible]

[illegible]

$A = (x+12)(y+16)$ [illegible] $xy = 972$ [illegible]

$\therefore A(x) = (x+12)\left(\frac{972}{x} + 16\right)$

$A'(x) = (x+12)\left(\frac{-972}{x^2}\right) + \left(\frac{972}{x} + 16\right)$

$A'(x) = \frac{-12 \times 972}{x^2} + 16$

$= \frac{16x^2 - 16 \times 27^2}{x^2}$

$= \frac{16(x^2 - 27^2)}{x^2}$

$A'(x) = 0$ [illegible] $x = \pm 27$

x [illegible] $x = 27$

[illegible] $A'(x) = \frac{16(x-27)(x+27)}{x^2}$

$0 < x < 27$ [illegible] $A'(x) < 0$

$27 < x$ [illegible] $A'(x) > 0$

$\therefore A(x)$ [illegible] $x = 27$ [illegible]

[illegible] $x = 27$ [illegible]

[illegible] $y = \frac{972}{27} = 36$

$\therefore$ [illegible] 39 cm × 52 cm [illegible]

06. (a) $I = \int_{11}^{23} \frac{1}{(x+1)\sqrt{2x+3}}\,dx$

$t = \sqrt{2x+3}$ ලෙස ගනිමු

එවිට $t^2 = 2x+3$ [illegible] $2t\,dt = 2\,dx$ [illegible]

[illegible] $x + 1 = \frac{t^2-3}{2} + 1 = \frac{t^2-1}{2}$ [illegible]

$x = 11$ [illegible] $t = 5$ [illegible] $x = 23$ [illegible] $t = 7$

[illegible]

$I = \int_5^7 \frac{2\,dt}{t^2-1} = \int_5^7 \frac{2\,dt}{(t-1)(t+1)}$

$= 2\int_5^7 \left[\frac{1/2}{t-1} + \frac{-1/2}{t+1}\right] dt$

$= \int_5^7 \frac{dt}{t-1} - \int_5^7 \frac{dt}{t+1}$

$= \left[\ln|t-1|\right]_5^7 - \left[\ln|t+1|\right]_5^7$

$= \ln\frac{6}{4} - \ln\frac{8}{6}$

$= \ln\frac{9}{8}$

(b) $J = \int e^{3x}\cos 4x\,dx$

$= \int e^{3x}\frac{d}{dx}\left[\frac{\sin 4x}{4}\right] dx$

$= e^{3x}\frac{\sin 4x}{4} - \int \frac{\sin 4x}{4}\frac{d}{dx}(e^{3x})\,dx$

$= e^{3x}\frac{\sin 4x}{4} - \int 3e^{3x}\frac{\sin 4x}{4}\,dx$

$= \frac{1}{4}e^{3x}\sin 4x - \frac{3}{4}\int e^{3x}\frac{d}{dx}\left(\frac{-\cos 4x}{4}\right) dx$

$= \frac{1}{4}e^{3x}\sin 4x - \frac{3}{4}\left(\frac{-e^{3x}\cos 4x}{4}\right) - \frac{3}{4}\int \frac{\cos 4x}{4}\frac{d}{dx}(e^{3x})\,dx$

$= \frac{1}{4}e^{3x}\sin 4x + \frac{3}{16}e^{3x}\cos 4x - \frac{3}{16}\int \cos 4x \cdot 3e^{3x}\,dx$

$= \frac{1}{4}e^{3x}\sin 4x + \frac{3}{16}e^{3x}\cos 4x - \frac{9}{16}\int e^{3x}\cos 4x\,dx$

$16J = 4e^{3x}\sin 4x + 3e^{3x}\cos 4x - 9J$

$25J = e^{3x}(4\sin 4x + 3\cos 4x)$

$J = \frac{e^{3x}}{25}(4\sin 4x + 3\cos 4x)$

$\therefore \int e^{3x}\cos 4x\,dx = \frac{e^{3x}}{25}(4\sin 4x + 3\cos 4x) + C$

මෙහි C අභිමත නියතයකි

(c) $\int \sin^4 2x\,dx = \int\left[\frac{1}{2}(1-\cos 4x)\right]^2 dx$

$= \frac{1}{4}\int (1 - 2\cos 4x + \cos^2 4x)\,dx$

$= \frac{1}{4}\int \left(1 - 2\cos 4x + \frac{1}{2}(1+\cos 8x)\right) dx$

$= \frac{1}{8}\int (3 - 4\cos 4x + \cos 8x)\,dx$

$= \frac{3}{8}x - \frac{4\sin 4x}{4 \times 8} + \frac{\sin 8x}{8 \times 8} + C$

$= \frac{3x}{8} - \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + C$

මෙහි C අභිමත නියතයකි

07. u [illegible] v [illegible] m [illegible]

එවිට [illegible]

$y = m(x-5)$ [illegible] $y = m(x+5)$ [illegible]

$\left.\begin{array}{l} y = mx - 5m \\ 3y + 4x = 25 \end{array}\right\} \Rightarrow x = \frac{25+15m}{4+3m}$ [illegible]

$y = \frac{5m}{4+3m}$

[illegible] $P = \left[\frac{25+15m}{4+3m}, \frac{5m}{4+3m}\right]$

[illegible]

$\left.\begin{array}{l} y = mx + 5m \\ 3y + 4x = 25 \end{array}\right\} \Rightarrow x = \frac{25-15m}{4+3m}$

[illegible] $y = \frac{45m}{4+3m}$

$Q = \left(\frac{25-15m}{4+3m}, \frac{45m}{4+3m}\right)$

එවිට $PQ^2 = \left(\frac{25+15m}{4+3m} - \frac{(25-15m)}{4+3m}\right)^2 + \left(\frac{5m}{4+3m} - \frac{45m}{4+3m}\right)^2$

$= \frac{900m^2}{(4+3m)^2} + \frac{1600m^2}{(4+3m)^2}$

$= \left(\frac{50m}{(4+3m)}\right)^2 = 5^2$

$\frac{10m}{4+3m} = \pm 1 \Rightarrow m = \frac{4}{7}$ [illegible] $m = -\frac{4}{13}$

m [illegible] u [illegible] v [illegible]

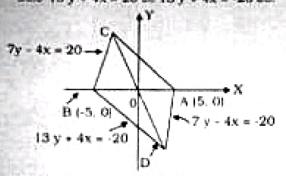
[illegible] $y = \frac{4}{7}(x-5)$ [illegible] $y = \frac{4}{7}(x+5)$

[illegible] $7y - 4x = -20$ [illegible] $7y - 4x = 20$

[illegible]

$y = -\frac{4}{13}(x-5)$ [illegible] $y = -\frac{4}{13}(x+5)$

[illegible] $13y + 4x = 20$ [illegible] $13y + 4x = -20$ [illegible]



[illegible] $y = 0$ [illegible] $(0, 0)$ [illegible]

[illegible]

$$7y - 4x + 20 + \lambda(13y + 4x + 20) = 0$$

[illegible]

$$\lambda = -1$$

[illegible] $3y + 4x = 0$

$$\left.\begin{array}{l} 7y - 4x = 20 \\ 3y + 4x = 0 \end{array}\right\} \Rightarrow y = 2 \text{ [illegible] } x = -\frac{3}{2}$$

$$\therefore C = \left(-\frac{3}{2}, 2\right)$$

ACBD [illegible]

$$= 2 \times ABC\,\Delta$$

$$= 2 \times \frac{1}{2} \times 2 \times 10$$

$$= 20 \text{ [illegible]}$$

08. $S_1 \equiv x^2 + y^2 - 2 = 0$

$S_2 \equiv x^2 + y^2 + 3x + 3y - 8 = 0$

$S_1 = 0$; [illegible] $C_1 \equiv (0, 0)$

[illegible] $r_1 = \sqrt{0 + 0 + 2} = \sqrt{2}$

$S_2 = 0$; [illegible] $C_2 = \left(-\frac{3}{2}, -\frac{3}{2}\right)$

[illegible] $r_2 = \sqrt{\frac{9}{4} + \frac{9}{4} + 8}$

$$= \frac{5\sqrt{2}}{2}$$

$$\therefore C_1C_2 = \sqrt{\frac{9}{4} + \frac{9}{4}} = \frac{3\sqrt{2}}{2}$$ [illegible]

$$r_2 - r_1 = \frac{5\sqrt{2}}{2} - \sqrt{2} = \frac{3\sqrt{2}}{2}$$

$r_2 - r_1 = C_1C_2$ [illegible]

$S_1 = 0$ [illegible] $S_2 = 0$ [illegible]

[illegible] (x_0, y_0) [illegible]

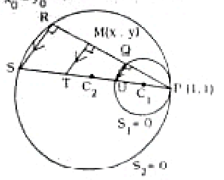
(x_0, y_0), $S_1 = 0$ [illegible] $S_2 = 0$ [illegible]

$$\therefore 3x_0 + 3y_0 - 6 = 0 \qquad \Rightarrow x_0 + y_0 = 2$$

[illegible] C_1 [illegible] C_2 [illegible] $y = x$ [illegible]

$$\Rightarrow y_0 = x_0$$

$\therefore x_0 = y_0 = 1$ [illegible] $P \equiv (1, 1)$ [illegible]



P, C_1, C_2 [illegible] U [illegible] S [illegible] $S_1 = 0$ [illegible] $S_2 = 0$ [illegible]

$S \equiv (x_1, y_1)$ [illegible]

[illegible] $\frac{x_1 + 1}{2} = -\frac{3}{2}$ [illegible] $\frac{y_1 + 1}{2} = -\frac{3}{2}$ [illegible]

$x_1 = -4,\ y_1 = -4$ [illegible] $\Rightarrow S \equiv (-4, -4)$

[illegible] $U \equiv (-1, -1)$ [illegible] SU [illegible] T [illegible]

$$T \equiv \left(\frac{-4 + -1}{2}, \frac{-4 + -1}{2}\right) = \left(\frac{-5}{2}, \frac{-5}{2}\right)$$

QR [illegible] M (x, y) [illegible]

TP [illegible] M [illegible]

TP [illegible] $= \left(\frac{-3}{4}, \frac{-3}{4}\right)$

$$\left(x + \frac{3}{4}\right)^2 + \left(y + \frac{3}{4}\right)^2$$

$$= \left(\frac{-3}{4} - 1\right)^2 + \left(\frac{-3}{4} - 1\right)^2$$

$$= \left(-\frac{7}{4}\right)^2 + \left(-\frac{7}{4}\right)^2$$

$$\Rightarrow x^2 + y^2 + \frac{3}{2}x + \frac{3}{2}y + \frac{9}{8} = \frac{49}{8}$$

[illegible] $x^2 + y^2 + \frac{3}{2}x + \frac{3}{2}y - 5 = 0$

$\therefore$ M [illegible]

$$x^2 + y^2 + \frac{3}{2}x + \frac{3}{2}y - 5 = 0$$

09. (a) $\sin\theta \tan\theta - 2(1 - \cos\theta)$

$$= \frac{\sin^2\theta - 2\cos\theta(1 - \cos\theta)}{\cos\theta}$$

$$= \frac{\cos^2\theta - 2\cos\theta + 1}{\cos\theta}$$

$$= \frac{(\cos\theta - 1)^2}{\cos\theta} > 0 \quad \because 0 < \theta < \frac{\pi}{2}$$

(b) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\frac{1 - \cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x\cos x} = \tan x$$

$$\tan\frac{\pi}{24} = \frac{1 - \cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}$$

$$= \frac{1 - \frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$= \frac{(4 - \sqrt{6} - \sqrt{2})}{(\sqrt{6} - \sqrt{2})} \times \frac{(\sqrt{6} + \sqrt{2})}{(\sqrt{6} + \sqrt{2})}$$

$$= \frac{4\sqrt{6} - 6 - \sqrt{12} + 4\sqrt{2} - \sqrt{12} - 2}{4}$$

$$= \frac{4\sqrt{6} + 4\sqrt{2} - 2\sqrt{12} - 8}{4}$$

$$= \frac{\cancel{4}(\sqrt{6} + \sqrt{2} - \sqrt{3} - 2)}{\cancel{4}}$$

$$= \sqrt{6} + \sqrt{2} - \sqrt{3} - 2$$

(c) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}$$

$\therefore a = \lambda\sin A,\ b = \lambda\sin B,\ c = \lambda\sin C$

$$\frac{a^2 - b^2}{c^2} = \frac{\lambda^2\sin^2 A - \lambda^2\sin^2 B}{\lambda^2\sin^2 C}$$

$$= \frac{\sin^2 A - \sin^2 B}{\sin^2(A + B)}$$

$$= \frac{(\sin A + \sin B)(\sin A - \sin B)}{\sin^2(A + B)}$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \cdot 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\sin^2(A + B)}$$

$$= \frac{2\sin\frac{A+B}{2}\cos\frac{A+B}{2} \cdot 2\sin\frac{(A-B)}{2}\cos\frac{(A-B)}{2}}{\sin^2(A + B)}$$

$$= \frac{\sin(A + B) \cdot \sin(A - B)}{\sin^2(A + B)}$$

$$= \frac{\sin(A - B)}{\sin(A + B)}$$

*** ***

Visit EDUHUB