

61. (i) $\alpha + \beta = 2$ සහ $\alpha\beta = 9$ වේ.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2 \times 9 = 4 - 18 = -14$$

$$\alpha^2 - 1 = (\alpha^2 + \beta^2) - 1 = -14 - 1 = -15$$

$$\gamma + \delta = (\alpha^2 - 1) + (\beta^2 - 1) = -15 - 1 = -16$$

$$\alpha^2 + \beta^2 = 2$$

$$= (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2 \times 9 = 4 - 18 = -14$$

$$= (-14)^2 - 2 \times 9 = 196 - 18 = 178$$

$$\gamma\delta = (\alpha^2 - 1)(\beta^2 - 1) = \alpha^2\beta^2 - \alpha^2 - \beta^2 + 1 = 9 - (-14) + 1 = 24$$

$$= \alpha^2\beta^2 - (\alpha^2 + \beta^2) + 1 = 9 - 2 + 1 = 8$$

$$= 81 - (-14) + 1 = 96$$

$$\therefore \text{අවසාන පිටුවේ පිටුවක}$$

$$[x - y] (x + y) = 0$$

$$\Rightarrow x^2 - (y + 8)x + y = 0$$

$$x^2 + 16x + 96 = 0$$

$$\text{ii) } f(x) = k = x^2 + 2x + 9 = k$$

$$\Rightarrow x^2 + 2x + 9 - k = 0$$

$$\Delta = 0$$

$$4 - 4(9 - k) = 0$$

$$\Rightarrow 9 - k = 1 \Rightarrow k = 8$$

$$\text{iii) } \frac{1}{f(x)} = \frac{1}{x^2 + 2x + 9} = \frac{1}{(x + 1)^2 + 8}$$

$$\text{අවම වශයෙන් } x + 1 = 0$$

$$\text{එනම් } x = -1$$

$$\therefore \text{විචලනය වන } = \frac{1}{8}$$

$$\text{iv) } f(x) = \lambda x \text{ සඳහා } x^2 + 2x + 9 = \lambda x$$

$$\Rightarrow x^2 + (2 - \lambda)x + 9 = 0$$

$$\Delta = 0$$

$$(2 - \lambda)^2 - 36 < 0 \text{ වේ.}$$

$$\text{එනම් } (\lambda + 4)(8 - \lambda) > 0$$

$$\therefore \text{අවසාන } x \text{ හි අගය}$$

$$= \{ \lambda \in \mathbb{R} : -4 < \lambda < 8 \}$$

62. (a) සමීකරණයේ සඳහා වන පරාමිති

$$= {}^{12}C_4$$

$$= \frac{12!}{4!8!}$$

$$= 476$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 495$$

(ii) අනුපාතය සමාන වන අනුපාතයේ අගය

$$= \frac{10!}{2!8!} = 45$$

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එකම වශයෙන් x හි අගය $\frac{1}{x}$ හි අගයට

සමාන වේ.

iii) $\frac{1}{x}$ අගය වන සඳහා

$$2x - 13 = -1 \text{ වේ. එනම් } x = 6 \text{ වේ.}$$

$$\text{එවිට } \frac{1}{x} \text{ හි අගය}$$

$$= {}^{13}C_6 (1 - 1)^0 \left(\frac{2}{6}\right)^{13-6}$$

$$= \frac{13!}{7!6!} \times 1 \times \frac{2^7}{6^7}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{2^7}{6^7}$$

$$= 13 \times 11 \times 2 \times 7 = 2002$$

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$$c = \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{1}{k!}$$

$$= \lim_{n \rightarrow \infty} \left[1 + \sum_{k=1}^n \frac{1}{k!} \right]$$

$$= 1 + \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k!}$$

$$\leq 1 + \lim_{n \rightarrow \infty} \left[2 - \frac{1}{2^{n-1}} \right]$$

$$= 1 + 2 - \lim_{n \rightarrow \infty} \left[\frac{1}{2^{n-1}} \right]$$

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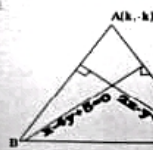
$$\leq 3$$

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ක) $\frac{5x-4}{(2+x)(1-x+x^2)} = \frac{A}{2+x} + \frac{Bx+C}{1-x+x^2}$
 $\Rightarrow A = -2, B = 2, C = -1$
 $\therefore \int \frac{5x-4}{(1-x+x^2)(2+x)} dx$
 $= \int \frac{2x-1}{1-x+x^2} dx - 2 \int \frac{dx}{x+2}$
 $= \ln |1-x+x^2| - 2 \ln |x+2|$
 $= \ln 3 - \ln 1 - 2(\ln 4 - \ln 3)$
 $= 3 \ln 3 - 2 \ln 4$
 $= \ln \frac{27}{16}$

මේ, $u_1 = 0 \Rightarrow u_2 = 0$ වීමෙන් පසුව පළමු පියවරයෙන් ලැබෙන $P(x_0, y_0)$ යනු පහත වන්නේය.
 $a_1x_0 + b_1y_0 - c_1 = 0 \Rightarrow 0$ සහ
 $a_2x_0 + b_2y_0 - c_2 = 0 \Rightarrow 0$
 එනම් $u_1 + u_2 = a_1x + b_1y + c_1 = 0$ සහ
 $a_2x + b_2y + c_2 = 0$ යන සමීකරණ දෙකේ
 සමඛාසීය ලක්ෂණයන් ඇති බව පෙන්වා දීමට
 $(a_1 + ka_2)x + (b_1 + kb_2)y + (c_1 + kc_2) = 0$
 හි k හි සුදුසු අගය තෝරා ගෙන එය සමීකරණයෙන්
 ඉවත් කළ විට $u_1 = 0$ සහ $u_2 = 0$ බව පෙන්වා දීමට
 හැකි වේ.



AB, $2x - y + 3 = 0$ සමාසීය වීමට
 AB, $x + 2y + p = 0$ සමාසීය වීමට
 සහ A(k, -k) සමාසීය වීමට
 $k - 2k + p = 0 \Rightarrow p = k$
 $\therefore$ AB හි සමාසීය $x + 2y + k = 0$ වේ.
 AC, $x - 4y + 5 = 0$ සමාසීය වීමට
 AC හි සමාසීය $4x + y + q = 0$ සමාසීය වේ.
 A(k, -k) සමාසීය වීමට
 $4k - k + q = 0 \Rightarrow q = -3k$
 $\therefore$ AC හි සමාසීය $4x + y - 3k = 0$ වේ.
 $2x - y + 3 = 0$ සහ $4x + y - 3k = 0$ හි
 සමාසීය වීමට
 $x = \frac{k-1}{2}, y = k-2$
 $x - 4y + 5 = 0 \Rightarrow x + 2y + k = 0$ හි
 සමාසීය වීමට
 $k = \frac{2k+5}{3} \Rightarrow \frac{5-k}{6}$

ඉහත සමාසීය සමීකරණ G = (x, y) වේ.
 $\bar{x} = \frac{1}{3} \left[k + \left(\frac{2k+5}{3} \right) + \left(\frac{k-1}{2} \right) \right]$
 $= \frac{5k-4k-10+3k-3}{18}$
 $= \frac{5k-13}{18}$
 $\bar{y} = \frac{1}{3} \left[k + \left(\frac{5-k}{6} \right) + (k+2) \right]$
 $= \frac{-7k+5-k+6k+12}{18}$
 $= \frac{17-k}{18}$
 $\bar{x} + 5\bar{y} = \frac{5k-13}{18} + \frac{5(17-k)}{18}$
 $= \frac{1}{18} (5k - 13 + 85 - 5k)$
 $= \frac{72}{18} = 4$

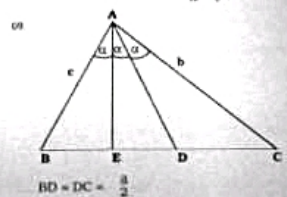
$\therefore G, x + 5y - 4 = 0$ සමාසීය වීමට වේ.
 එනම් G යනු C_1 හි සමාසීය වීමට වේ.
 C_2 හි සමාසීය වීමට $C_1C_2 = |r_1 + r_2|$ වීමට වේ.
 එනම් r_1 හා r_2 යනු සමාසීය වීමට වේ.

සමාසීය
 $\sqrt{r_1^2 + r_2^2} = \sqrt{r_1^2 + r_2^2}$
 $= \sqrt{r_1^2 + r_2^2 + c_1^2 + c_2^2} /$
 යනු (x_0, y_0) ලක්ෂණයක් සහිත වීමට
 ලක්ෂණය C_1C_2 වීමට වේ.
 C_1C_2 හි සමාසීය
 $y - (-1) = \frac{(-1) - (-1)}{(-1) - (-1)} (x - (-1))$
 $y + 1 = \frac{(1 - (-1))}{(k_2 - k_1)} (x + 1)$
 එනම්
 $(k_2 - k_1)y + 1(k_2 - k_1) = (k_1 - k_2)x + 1(k_1 - k_2)$

එනම්,
 $(k_1 - k_2)x - (k_1 - k_2)y + 1(k_2 - k_1) = 0$
 එනම්
 $x_0^2 + y_0^2 + 2g_1x_0 + 2f_1y_0 + c_1 = 0$
 $x_0^2 + y_0^2 + 2g_2x_0 + 2f_2y_0 + c_2 = 0$
 $\Rightarrow 2(g_1 - g_2)x_0 + 2(f_1 - f_2)y_0 + c_1 - c_2 = 0$
 එනම් (x_0, y_0)
 $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$
 වීමට වේ.
 එනම් සමාසීය ලක්ෂණයන් සහිත ලක්ෂණය
 $2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0$ වීමට වේ.
 එනම් G වේ.

සමාසීය ලක්ෂණයන් සහිත ලක්ෂණය
 $x^2 + y^2 - 2x + 4y = 0$ සහ
 $x^2 + y^2 - 10x + 20 = 0$ සහ $5C$
 $C_1C_2 = \sqrt{(1-1)^2 + (2-0)^2}$
 $= \sqrt{20} = 2\sqrt{5}$
 $r_1 = \sqrt{(1-1)^2 + 2^2} = \sqrt{5}$
 $r_2 = \sqrt{(1-5)^2 + 2^2} = \sqrt{5}$

$r_1 + r_2 = 2\sqrt{5}$
 $C_1C_2 = r_1 + r_2$
 $\therefore$ සමාසීය ලක්ෂණයන් සහිත වේ.
 සමාසීය ලක්ෂණයන් සහිත වීමට C_1C_2 හි සමාසීය
 ලක්ෂණයන් සහිත වේ.
 $\therefore$ සමාසීය ලක්ෂණය $A = \left[\frac{1-5}{2}, \frac{(2+0)}{2} \right]$
 $= (3, -1)$
 $P(x, y)$ යනු සමාසීය ලක්ෂණයන් සහිත වේ.
 එනම්, P හි සමාසීය ලක්ෂණයන් සහිත වීමට
 $\sqrt{x^2 + y^2} - 2x + 4y = 0$ සහ $\sqrt{x^2 + y^2} - 10x + 20 = 0$ වේ.
 $\sqrt{x^2 + y^2} - 2x + 4y = 0$
 $= k \sqrt{x^2 + y^2} - 10x + 20$
 $\Rightarrow x^2 + y^2 - 2x + 4y = k^2(x^2 + y^2) - 10kx + 20k$
 $\Rightarrow (k^2 - 1)x^2 + (k^2 - 1)y^2 - (10k - 2)x - 4y + 20k^2 = 0$
 $(x^2 + y^2)(k^2 - 1) - (10k - 2)x - 4y + 20k^2 = 0$
 $k^2 - 1 = 0$ සහ P හි සමාසීය ලක්ෂණයන්
 සහිත වීමට
 $x^2 + y^2 = \frac{(10k-2)x - 4y + 20k^2}{k^2 - 1}$
 $= \frac{20k^2}{k^2 - 1} = 0$



ABD Δ 1000 1000 1000

$$\frac{BD}{\sin 2\alpha} = \frac{AB}{\sin \hat{ADB}}$$

$$\Rightarrow \sin \hat{ADB} = \frac{AB}{BD} \sin 2\alpha$$

ADC Δ 1000 1000 1000

$$\frac{AC}{\sin \hat{ADC}} = \frac{DC}{\sin \alpha}$$

$$\Rightarrow \sin \hat{ADC} = \frac{AC}{DC} \sin \alpha$$

$$\sin \hat{ADB} = \sin \hat{ADC}$$

$$\Rightarrow \frac{AB}{DC} \sin 2\alpha = \frac{AC}{DC} \sin \alpha$$

$$\Rightarrow AC \sin \alpha = AB \cdot 2 \sin \alpha \cos \alpha$$

(∵ DC = BD 1000)

$$\Rightarrow \cos \alpha = \frac{AC}{2AB} = \frac{b}{2c}$$

$$\Rightarrow \cos \frac{A}{2} = \frac{b}{2c}$$

$$DE : EB = 1 : k \Rightarrow DE = \frac{1}{1+k} BD \Rightarrow$$

$$EB = \frac{k}{1+k} BD$$

$$\Rightarrow DE = \frac{a}{2(1+k)} \Rightarrow EB = \frac{ak}{2(1+k)}$$

1000

$$EC = DE + DC = \frac{a(2+k)}{2(1+k)}$$

AEC Δ 1000 1000 1000

$$\frac{EC}{\sin 2\alpha} = \frac{AC}{\sin \hat{AEC}}$$

$$\Rightarrow \sin \hat{AEC} = \frac{AC}{EC} \sin 2\alpha$$

1000 AEB Δ 1000

$$\frac{BE}{\sin \alpha} = \frac{AB}{\sin \hat{AEB}}$$

$$\Rightarrow \sin \hat{AEB} = \frac{AB}{BE} \sin \alpha$$

$$\sin \hat{AEC} = \sin \hat{AEB}$$

$$\Rightarrow \frac{AC}{EC} \sin 2\alpha = \frac{AB}{BE} \sin \alpha$$

$$\Rightarrow \cos \alpha = \frac{1}{2} \frac{AB}{BE} \frac{EC}{AC}$$

$$= \frac{1}{2} \frac{2c(1+k)}{ak} \frac{a(2+k)}{2b(1+k)}$$

$$= \frac{(2+k)c}{2bk}$$

$$k=1 \Rightarrow \cos \alpha = \frac{3c}{2b} = \frac{b}{2c} \Rightarrow b^2 = 3c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{3}{4}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 30^\circ$$

$$\therefore A = 3\alpha = 90^\circ$$

$$k=2 \Rightarrow \cos \alpha = \frac{4c}{4b} = \frac{b}{2c} \Rightarrow b^2 = 2c^2$$

$$\Rightarrow \left(\frac{b}{2c}\right)^2 = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{b}{2c} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

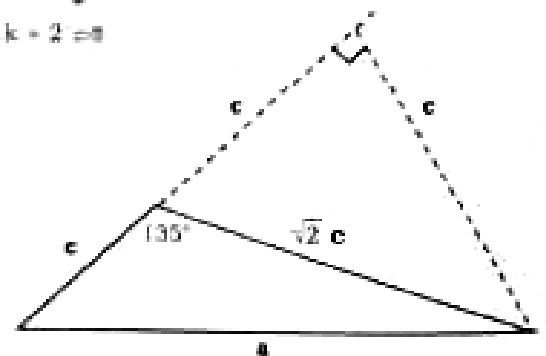
$$\therefore A = 135^\circ$$

$$k=1 \Rightarrow$$

$$a^2 = b^2 + c^2 = 3c^2 + c^2 = 4c^2 \Rightarrow c = \frac{a}{2}$$

$$b = \frac{\sqrt{3}}{2} a$$

$$k=2 \Rightarrow$$



$$a^2 = (AC)^2 + (AB)^2 + 2(AB)(AC) \cos 45^\circ$$

$$= 2c^2 + c^2 + 2\sqrt{2}c^2 \cdot \frac{1}{\sqrt{2}}$$

$$= 3c^2$$

$$\Rightarrow c^2 = \frac{a^2}{3}$$

$$\therefore c = \frac{a}{\sqrt{3}}, b = \sqrt{\frac{2}{3}} a$$
