1. (අ) $\alpha + \beta = p$ හා $\alpha\beta = q$

$\alpha(\alpha + \beta)$ හා $\beta(\alpha + \beta)$ මූල වන සමීකරණය $x^2 - ax + b = 0$ ලෙස ගනිමු.

එවිට $a = \alpha(\alpha + \beta) + \beta(\alpha + \beta)$

$= (\alpha + \beta)^2 = p^2$ හා

$b = [\alpha(\alpha + \beta)][\beta(\alpha + \beta)]$

$= \alpha\beta(\alpha + \beta)^2 = qp^2$

∴ අවශ්‍ය සමීකරණය

$x^2 - p^2x + qp^2 = 0$

(ආ) ∴ සියලු x, y සඳහා

$2x^2 + \lambda xy + 3y^2 - 5y - 2 =$

$(ax + by + c)(px + qy + r)$ ලෙස ගනිමු.

$x = 0$ විට $3y^2 - 5y - 2$

$= (by + c)(qy + r)$ සියලු y සඳහා

නමුත් $3y^2 - 5y - 2 = (3y+1)(y-2)$

∴ $b = 3, c = 1, q = 1$ හා $r = -2$

∴ සියලු x, y සඳහා

$2x^2 + \lambda xy + 3y^2 - 5y - 2 =$

$(ax + 3y + 1)(px + y - 2)$ වේ

දැන් $y = 0$ විට සියලු x සඳහා

$2x^2 - 2 = (ax + 1)(px - 2)$

අනුරූප සංගුණක සැසඳීමෙන්

$2 = ap$ හා $-2a + p = 0$

$a^2 = 1 \Rightarrow a = \pm 1$ ඒ අනුව

$p = \pm 2$ වේ

∴ $2x^2 + \lambda xy + 3y^2 - 5y - 2$

$= (-x + 3y + 1)(-2x + y - 2)$ හෝ

$= (x + 3y + 1)(2x + y - 2)$ හෝ වේ.

xy හි සංගුණක සැසඳීමෙන් $\lambda = \pm 7$ වේ.

(ඇ) $\dfrac{2x^3 - x + 3}{x(x-1)^2} = A + \dfrac{B}{x} + \dfrac{C}{x-1} + \dfrac{D}{(x-1)^2}$

මෙහි A, B, C හා D නියත වේ.

$$\frac{2x^3 - x + 3}{x(x-1)^2} = \frac{A(x^3-2x^2+x)+B(x^2-2x+1)+C(x^2-x)+Dx}{x(x-1)^2}$$

අනුරූප සංගුණක සැසඳීමෙන්

$A = 2,\ -2A + B + C = 0,$
$A - 2B - C + D = -1$ හා $B = 3$

එමනිසා $C = 1$ හා $D = 4$ වේ

∴ $\dfrac{2x^3 - x + 3}{x(x-1)^2} = 2 + \dfrac{3}{x} + \dfrac{1}{x-1} + \dfrac{4}{(x-1)^2}$

2. (අ) $U_n = 1.n + 2(n-1) + \ldots\ldots +$
$(n-1).2 + n.1$ යි

$U_n = \dfrac{1}{6}n(n+1)(n+2)$ —— ①

සාධනය

$n = 1$ විට

① හි L. H. S $= U_1 = 1.1 = 1$

① හි R. H. S. $= \dfrac{1}{6}.1.2.3 = 1$

∴ ① හි L.H.S = ① හි R.H.S

∴ $n = 1$ විට ප්‍රතිඵලය සත්‍ය වේ

$n = p$ විට ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනය කරමු

එවිට $U_p = \dfrac{1}{6}p(p+1)(p+2)$

∴ $n = p + 1$ විට U_{p+1}

$= 1.(p+1) + 2.p + \ldots\ldots + p.2 + (p+1).1$

$= 1.p + 2(p-1) + \ldots\ldots + p.1$
$+ (1 + 2 + \ldots\ldots + (p+1)).1$

$= U_p + (1 + 2 + \ldots\ldots + (p+1))$

$= \dfrac{1}{6}p(p+1)(p+2) + \dfrac{1}{2}(p+1)(p+2)$

$= \dfrac{1}{6}(p+1)(p+2)(p+3)$

∴ $n = p + 1$ විටද ප්‍රතිඵලය සත්‍ය වේ.
ගණිත අභ්‍යුහන මූලධර්මය අනුව
සියලු ධන නිඛිලමය n සඳහා

$U_n = \dfrac{1}{6}n(n+1)(n+2)$ වේ

සියලුම ධන නිඛිලමය n සඳහා

$\dfrac{1}{U_n} = \dfrac{6}{n(n+1)(n+2)}$

$= \dfrac{3}{n(n+1)} - \dfrac{3}{(n+1)(n+2)}$

$= V_n - V_{n+1}$

මෙහි $V_n = \dfrac{3}{n(n+1)}$

$\dfrac{1}{U_1} = V_1 - V_2$

$\dfrac{1}{U_2} = V_2 - V_3$

....

....

....

$\dfrac{1}{U_{n-1}} = V_{n-1} - V_n$

$\dfrac{1}{U_n} = V_n - V_{n+1}$

$$\sum_{r=1}^{n} \frac{1}{U_r} = V_1 - V_{n+1}$$

$$= \frac{3}{2} - \frac{3}{(n+1)(n+2)}$$

$$\sum_{r=1}^{\infty} \frac{1}{U_r} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{U_r}$$

$$= \lim_{n\to\infty} \left(\frac{3}{2} - \frac{3}{(n+1)(n+2)} \right)$$

$$= \frac{3}{2}$$

$$\because \lim_{n\to\infty} \frac{3}{(n+1)(n+2)} = 0$$

(ආ) $(1+kx)^{10}$

$$= {}^{10}C_0 + {}^{10}C_1 kx + {}^{10}C_2 k^2x^2 + \ldots + {}^{10}C_r k^r x^r + \ldots + C_{10} k^{10} x^{10}$$

$$= a_0 + a_1 x + a_2 x^2 + \ldots + a_{10} x^{10}$$

$$\therefore \sum_{r=0}^{10} {}^{10}C_r k^r x^r = \sum_{r=0}^{10} a_r x^r, \quad \forall x \in R$$

$\therefore {}^{10}C_r k^r = a_r,\ r = 0, 1, 2, \ldots 10$ [illegible]

${}^{10}C_2 k^2 = a_2 = \frac{20}{9}$ [illegible]

[illegible] ${}^{10}C_2 k^2 = \frac{10.9}{2} k^2$

$$\Rightarrow \frac{10.9}{2} k^2 = \frac{20}{9}$$

$$\therefore k^2 = \left(\frac{2}{9}\right)^2 \Rightarrow k = \frac{2}{9}$$

($\because$ k [illegible])

$$\therefore \left(1 + \frac{2}{9}x\right)^{10} = a_0 + a_1 x + a_2 x^2 + \ldots a_{10} x^{10}$$

$x = 1$ [illegible]

$$\left(\frac{11}{9}\right)^{10} = a_0 + a_1 + a_2 + \ldots + a_{10} \text{ ——— } ①$$

$x = -1$ [illegible]

$$\left(\frac{7}{9}\right)^{10} = a_0 - a_1 + a_2 - \ldots - a_9 + a_{10} \text{ ——— } ②$$

$$① - ② \quad \left(\frac{11}{9}\right)^{10} - \left(\frac{7}{9}\right)^{10} = 2\left(a_1 + a_3 + \ldots + a_9\right)$$

[illegible]

$$a_1 + a_3 + a_5 + a_7 + a_9 = \frac{11^{10} - 7^{10}}{2 \cdot 9^{10}} \text{ ——— } ③$$

② + ③

$$\Rightarrow a_0 + a_2 + a_4 + a_6 + a_8 + a_{10} = \left(\frac{7}{9}\right)^{10} + \frac{11^{10} - 7^{10}}{2 \times 9^{10}}$$

$$= \frac{2 \times 7^{10} + 11^{10} - 7^{10}}{2 \times 9^{10}}$$

$$= \frac{11^{10} + 7^{10}}{2 \times 9^{10}}$$

03. (අ) $\frac{(-1+i)^7}{(1+i)^4} = \frac{(-1+i)^7 (1-i)^4}{(1+i)^4 (1-i)^4}$

$$= \frac{(1-i)^7}{2^4}$$

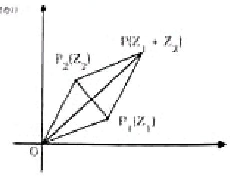
$$= -\frac{1}{2^4}(1 - 7i - 21 + 35i + 35 - 21i - 7 + i)$$

$$= -\frac{1}{2} - \frac{1}{2} i$$

$$= \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right)$$

$\therefore$ [illegible] $\frac{1}{\sqrt{2}}$ [illegible] $\frac{5\pi}{4}$

(ආ)



P_1, P_2 [illegible] Z_1 [illegible] Z_2 [illegible] OP_1 [illegible] OP_2 [illegible] OP_1PP_2 [illegible] P [illegible] $Z_1 + Z_2$ [illegible]

[illegible] -

$$Z_1 = x_1 + iy_1 \text{ [illegible] } Z_2 = x_2 + iy_2$$

[illegible]

[illegible] $P_1 = (x_1, y_1)$ [illegible] $P_2 = (x_2, y_2)$ [illegible]

$P = (x, y)$ [illegible]

[illegible] $\frac{x+0}{2} = \frac{x_1 + x_2}{2}$ [illegible]

$$\frac{y+0}{2} = \frac{y_1 + y_2}{2}$$

$\Rightarrow x = x_1 + x_2$ [illegible] $y = y_1 + y_2$

$P = (x_1 + x_2, y_1 + y_2)$

P [illegible] $(x_1 + x_2) + i(y_1 + y_2)$ [illegible]

[illegible] P [illegible] $(x_1 + iy_1) + (x_2 + iy_2)$ [illegible]

[illegible] P [illegible] $Z_1 + Z_2$ [illegible]

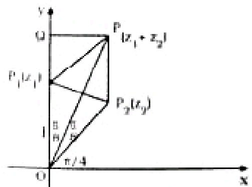
$$Z_1 = \frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)}$$

$$= \frac{1 + 2i + i^2}{1 - i^2} = \frac{2i}{2} = i$$

$$Z_2 = \frac{\sqrt{2}}{1-i} = \frac{1}{\sqrt{2}}(1+i)$$

$$= 1(\cos \pi/4 + i \sin \pi/4)$$

$$Z_1 + Z_2 = \frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}} + 1\right) i$$



$$\tan \frac{5}{8} = \frac{QP}{OQ}$$

$$= \frac{Re(z_1 + z_2)}{Im(z_1 + z_2)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + 1}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1+\sqrt{2}}{\sqrt{2}}} = \frac{1}{1+\sqrt{2}}$$

$$= \frac{1-\sqrt{2}}{1-2} = \sqrt{2} - 1$$

04. (අ) $\lim_{x\to 0} \frac{1 - \cos^2(2\sin x)}{2\sin^2 x}$

$$= \lim_{x\to 0} \frac{2\sin^2(2\sin x)}{(2\sin x)^2}$$

$$= \lim_{x\to 0} 2\left[\frac{\sin(2\sin x)}{2\sin x}\right]^2$$

$$= 2\lim_{t\to 0} \left(\frac{\sin t}{t}\right)^2$$

$= 2$ [illegible] $t = 2\sin x$ $x \to 0$ [illegible] $t \to 0$ [illegible]

(ආ) $y = e^{k\sin^{-1}x}$

$$\frac{dy}{dx} = e^{k\sin^{-1}x} \times k \times \frac{1}{\sqrt{1-x^2}}$$

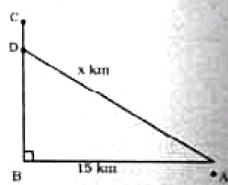
$$\sqrt{1-x^2}\,\frac{dy}{dx} = k e^{k\sin^{-1}x}$$

$$= ky$$

$x = \frac{1}{2}$ [illegible] $y = e^{k\pi/6}$

[illegible] $\frac{dy}{dx} = \frac{2}{\sqrt{3}} k e^{k\pi/6}$

(ඇ)



$DC = 50 - \sqrt{x^2 - 225}$; $x \geq 15$ [illegible]

$$T(x) = \frac{x}{40} + 1 - \frac{\sqrt{x^2 - 225}}{50}$$

$$\frac{dT(x)}{dx} = \frac{1}{40} - \frac{1}{2} \times \frac{2x}{50\sqrt{x^2 - 225}}$$

$$\frac{dT(x)}{dx} = \frac{1}{40} - \frac{x}{50\sqrt{x^2 - 225}}$$

$$= \frac{1}{200\sqrt{x^2 - 225}} \times \left\{5\sqrt{x^2 - 225} - 4x\right\}$$

$= 0 \quad x = 25$ [illegible]

x හි අගය	$15 < x < 25$	$x = 25$	$25 < x < 50$
$\frac{dT(x)}{dx}$	(-)	0	(+)

x හි අගය 15 සිට 25 දක්වා [illegible] T(x) [illegible]

x හි අගය 25 සිට 50 දක්වා [illegible] T(x) [illegible]

x හි අගය 25 විට T(x) [illegible]

∴ [illegible] T(25) වේ

එනම් [illegible] $= \frac{25}{40} + 1 - \frac{\sqrt{625 - 225}}{50}$

$= \frac{5}{8} + 1 - \frac{20}{50}$

$= 0.625 + 1 - 0.400$

$= \underline{1.225}\ h$

05. (අ) $I = \int_1^8 \frac{1}{x^{\frac{1}{3}} + x^{\frac{2}{3}}}\, dx$

$y = x^{\frac{1}{3}}$ ලෙස ගනිමු. එවිට $dy = \frac{1}{3}\frac{1}{x^{\frac{2}{3}}}\, dx$

එනම් $3y^2 dy = dx$

$x = 1$ විට $y = 1$, $x = 8$ විට $y = 2$

එවිට $I = 3\int_1^2 \frac{dy}{1 + y^2} = 3\left[\tan^{-1} y\right]_1^2$

$= 3\left\{\tan^{-1} 2 - \tan^{-1} 1\right\}$

$= 3\left\{\tan^{-1} 2 - \pi/4\right\}$

(ආ) $I = \int_0^{\pi} e^{-2x} \cos x\, dx$

$= \int_0^{\pi} e^{-2x} \frac{d}{dx}(\sin x)\, dx$

$= \left[e^{-2x} \underset{0}{\sin x}\right]_0^{\pi} + 2\int_0^{\pi} e^{-2x} \sin x\, dx$

$= 2J$ —— ①

$J = \int_0^{\pi} e^{-2x} \sin x\, dx$

$= \int_0^{\pi} e^{-2x} \frac{d}{dx}(-\cos x)\, dx$

$= \left[-e^{-2x} \cos x\right]_0^{\pi} - 2\int_0^{\pi} e^{-2x} \cos x\, dx$

$= e^{-2\pi} + 1 - 2I$ —— ②

① හා ② න් $I = \frac{1}{5}(e^{2\pi} + 1)$

$I = \frac{2}{5}(e^{2\pi} + 1)$

(ඇ) $\frac{x^2 - 5x}{(x - 1)(x + 1)^2}$

$= \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$

මෙහි A, B හා C නියත වේ

∴ $A = -1$ හා $C = -3$

$x = 0$ විට $0 = 1 + B - 3 \Rightarrow B = 2$

$\int \frac{x^2 - 5x}{(x - 1)(x + 1)^2}\, dx$

$= -\int \frac{dx}{(x - 1)} + 2\int \frac{dx}{x + 1} - 3\int \frac{dx}{(x + 1)^2}$

$= -\ln|x - 1| + 2\ln|x + 1| + \frac{3}{x + 1} + D$

D යනු අභිමත නියතයකි

06. [illegible] සමීකරණය $y = mx + c$ ලෙස ගනිමු

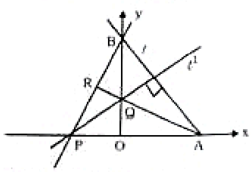
$(a, 0)$ හා $(0, b)$ ලක්ෂ්‍ය [illegible]

$0 = am + c$ හා $b = c$ වේ

∴ $m = -\frac{b}{a}$

∴ [illegible] සමීකරණය

$y = -\frac{b}{a}x + b \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$



l හි සමීකරණය $\frac{x}{h} + \frac{y}{k} = 1$ [illegible]

$A = (h, 0)$ හා $B = (0, k)$

l^1 හි සමීකරණය

$\frac{x}{h^1} + \frac{y}{k^1} = 1$ ලෙස ගනිමු. $l \perp l^1$ [illegible]

$\left(-\frac{k^1}{h^1}\right)\left(-\frac{k}{h}\right) = -1 \Rightarrow \frac{k^1}{h} = \frac{-h^1}{k} = t$ ලෙස ගනිමු

එවිට $k^1 = ht$, $h^1 = -kt$ හා

එමගින් $P = (-kt, 0)$ හා $Q = (0, ht)$ ලෙස ලැබේ

AQ හි සමීකරණය $\frac{y - 0}{x - h} = \frac{0 - ht}{h - 0} = -t$

$\Rightarrow y = -tx + ht$

$y + tx = ht$

BP හි සමීකරණය $\frac{y - k}{x - 0} = \frac{k - 0}{0 - (-kt)} = \frac{1}{t}$

$\Rightarrow yt - kt = x$

$\Rightarrow yt - x = kt$

$R \equiv (x_0, y_0)$ නම් එවිට $y_0 = t(h - x_0)$ හා

$x_0 = t(y_0 - k)$

∴ $\frac{y_0}{h - x_0} = \frac{x_0}{y_0 - k}$, $x_0 \neq h$ හා $y_0 \neq k$ විට

එනම් $x_0^2 + y_0^2 - hx_0 - ky_0 = 0$

එමගින් R ලක්ෂ්‍යය $x^2 + y^2 - hx - ky = 0$ වෘත්තය [illegible]

07. C_1 හා C_2 වෘත්ත [illegible] r_1 හා r_2 [illegible] ලෙස ගනිමු

එවිට $C_1 \equiv (-g_1, -f_1)$; $r_1 = \sqrt{g_1^2 + f_1^2 - c_1}$ හා

$C_2 \equiv (-g_2, -f_2)$; $r_2 = \sqrt{g_2^2 + f_2^2 - c_2}$

වෘත්ත දෙක [illegible]

$(C_1C_2)^2 = r_1^2 + r_2^2$

එනම් $(g_1 - g_2)^2 + (f_1 - f_2)^2$

$= (g_1^2 + f_1^2 - c_1) + (g_2^2 + f_2^2 - c_2)$

එනම් $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$(p, 0)$ [illegible] r [illegible] ලෙස ගනිමු. එවිට වෘත්තයේ සමීකරණය

$(x - p)^2 + y^2 = r^2$

එනම් $x^2 + y^2 - 2px + p^2 - r^2 = 0$

$x^2 + y^2 - 8x - 6y + 21 = 0$ වෘත්තය [illegible]

[illegible]

$2(-p)(-4) = 21 + p^2 - r^2$

එනම් $r^2 = p^2 - 8p + 21$ —— ❶

තවද $x^2 + y^2 + 4x + 6y + 9 = 0$ වෘත්තය [illegible]

$|\pm r + 2| = \sqrt{(p - (-2))^2 + (0 - (-3))^2}$

$= \sqrt{(p + 2)^2 + 3^2}$ —— ❷

$r^2 \pm 4r + 4 = p^2 + 4p + 4 + 9$

$r^2 \pm 4r = p^2 + 4p + 9$ —— ❸

❸ - ❶ ⇒

$r = \pm 3(p - 1)$ —— ❹

එවිට,

❶ න් $9p^2 - 18p + 9 = p^2 - 8p + 21$

$8p^2 - 10p - 12 = 0$

$4p^2 - 5p - 6 = 0$

$(4p + 3)(P - 2) = 0$

$p = -¾$ හෝ $p = 2$

$P = 2$ විට ❹ [illegible] $r = 3$

$P = -3/4$ විට ❹ [illegible]

$r = \frac{21}{4}$

∴ $P = 2$, $r = 3$ විට ❷ [illegible]

$3 + 2 = \sqrt{(2 + 2)^2 + 9} = 5$

[illegible]

[illegible]

∴ [illegible]

එවිට වෘත්තයේ සමීකරණය

$(x - 2)^2 + y^2 = 9$

එනම් $x^2 + y^2 - 4x - 5 = 0$

$P = -\frac{3}{4}$ හා $r = 21/4$ විට ❷ [illegible]

$\left|-\frac{21}{4} + 2\right| = \sqrt{\left(-\frac{3}{4} + 2\right)^2 + 9}$

$\frac{13}{4} = \sqrt{\frac{25}{16} + \frac{144}{16}}$

∴ [illegible]

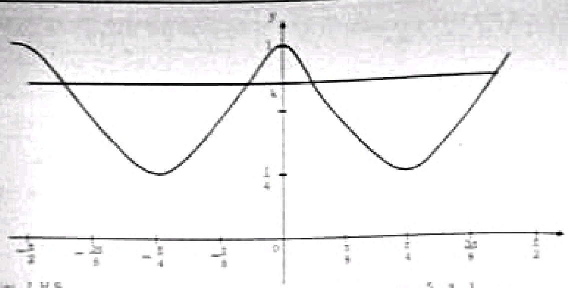
[illegible]

∴ [illegible]

∴ එවිට වෘත්තයේ සමීකරණය

$(x + ¾)^2 + y^2 = \left[\frac{21}{4}\right]^2$

එනම් $2x^2 + 2y^2 + 3x - 54 = 0$



(ii) L.H.S

$= \dfrac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta}$

$= \dfrac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta}$

$= \dfrac{(1 + \sin\theta + \cos\theta)^2}{(1 + \sin\theta)^2 - \cos^2\theta}$

$= \dfrac{(1+\sin\theta)^2 + 2\cos\theta(1+\sin\theta) + \cos^2\theta}{1 + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \dfrac{(1+\sin\theta)^2 + 2\cos\theta(1+\sin\theta) + (1+\sin\theta)(1-\sin\theta)}{2\sin\theta(1+\sin\theta)}$

$= \dfrac{1 + \sin\theta + 2\cos\theta + (1 - \sin\theta)}{2\sin\theta}$

$= \dfrac{2(1 + \cos\theta)}{2\sin\theta} = \dfrac{1 + \cos\theta}{\sin\theta}$

(iii) $8(\cos^6 x + \sin^6 x)$

$= 8\{\sin^6 x + (1 - \sin^2 x)^3\}$

$= 8(\sin^6 x + 1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x)$

$= 8\{1 - 3\sin^2 x + 3\sin^4 x\}$

$= 8 - 24\sin^2 x(1 - \sin^2 x)$

$= 8 - 6\sin^2 2x$

$= 8 - 3(1 - \cos 4x)$

$= 5 + 3\cos 4x$

$y = \cos^6 x + \sin^6 x$ [illegible]

$y = \frac{5}{8} + \frac{3}{8}\cos 4x$ [illegible]

y [illegible] $= \frac{8}{8} = 1$ [illegible]

$\cos 4x = 1 \Rightarrow x = 0; -\pi/2 \le x \le \pi/2$

y [illegible] $= \frac{5}{8} - \frac{3}{8} = \frac{1}{4}$ [illegible]

$\cos 4x = -1$

$\Rightarrow x = \pm\pi/4 ; -\pi/2 \le x \le \pi/2$

(i) $k < \frac{1}{4}$ [illegible] $k > 1$ [illegible]

$y = \cos^6 x + \sin^6 x$ [illegible] $y = k$ [illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(ii) $k = \frac{1}{4}$ [illegible] $y = k$ [illegible]

$y = \cos^6 x + \sin^6 x$ [illegible]

$x = -\pi/4$ [illegible] $x = \pi/4$ [illegible]

$\cos^6 x + \sin^6 x = k$ [illegible]

(iii) $k = 1$ [illegible] $y = k$ [illegible] $y = \cos^6 x + \sin^6 x$ [illegible] $x = -\pi/2,\ 0$ [illegible] $\pi/2$ [illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(iv) $\frac{1}{4} < k < 1$ [illegible] $y = \cos^6 x + \sin^6 x$

[illegible] $y = k$ [illegible]

[illegible] $\cos^6 x + \sin^6 x = k$ [illegible]

(b) $4\sin^2 x + 12\sin x \cos x - \cos^2 x + 5 = 0$

$2(1 - \cos 2x) + 6\sin 2x - \dfrac{\cos 2x + 1}{2} + 5 = 0$

$\Rightarrow 12\sin 2x - 5\cos 2x + 13 = 0$

$\frac{12}{13}\sin 2x - \frac{5}{13}\cos 2x = -1$

$\frac{5}{13}\cos 2x - \frac{12}{13}\sin 2x = 1$

$\cos\alpha = 5/13,\ \sin\alpha = 12/13$ [illegible]

$\cos\alpha, \sin\alpha > 0$ [illegible] $0 < \alpha < \pi/2$

[illegible] $\cos\alpha \cos 2x - \sin\alpha \sin 2x = 1$

[illegible] $\cos(2x + \alpha) = 1$

$2x + \alpha = 2n\pi \qquad n \in \mathbb{Z}$

[illegible] $x = n\pi - \alpha/2$

$n = 1$ [illegible] $x = \pi - \alpha/2$ [illegible]

$0 < \pi - \alpha/2 < 2\pi$

$n = 2$ [illegible] $x = 2\pi - \alpha/2$ [illegible]

$0 < 2\pi - \alpha/2 < 2\pi$

[illegible] $x = \pi - \alpha/2 \Rightarrow x = 2\pi - \alpha/2$ [illegible] $0 < x < 2\pi$ [illegible]

(c) [illegible] $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

[illegible] $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\dfrac{b + c}{2k - 1} = \dfrac{c + a}{2k} = \dfrac{a + b}{2k + 1} = \lambda$ [illegible]

$b + c = \lambda(2k - 1)$ — ❶
$c + a = 2\lambda k$ — ❷
$a + b = \lambda(2k + 1)$ — ❸

❶ ❸ [illegible]

$b - a = -\lambda$ — ❹

❷ ❸ [illegible]

$c - b = -\lambda$ — ❺

❹ + ❺

$c - a = -2\lambda$ — ❻

$c + a = 2\lambda k \quad \therefore 2c = 2\lambda(k - 1)$

$\dfrac{c}{k - 1} = \lambda$

❶ + ❸

$2b = 2\lambda k \Rightarrow \dfrac{b}{k} = \lambda$

❷ - ❻

$2a = 2\lambda k + 2\lambda \Rightarrow \dfrac{a}{1 + k} = \lambda$

[illegible] $\dfrac{a}{k + 1} = \dfrac{b}{k} = \dfrac{c}{k - 1}$

$\Rightarrow \dfrac{\sin A}{k + 1} = \dfrac{\sin B}{k} = \dfrac{\sin C}{k - 1}$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{k^2 + (k - 1)^2 - (k + 1)^2}{2k(k - 1)}$

$= \dfrac{k - 4}{2(k - 1)}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$= \dfrac{(k + 1)^2 + (k - 1)^2 - k^2}{2(k + 1)(k - 1)}$

$= \dfrac{k^2 + 2}{2(k^2 - 1)}$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{(k + 1)^2 + k^2 - (k - 1)^2}{2k(k + 1)}$

$= \dfrac{k + 4}{2(k + 1)}$

$\dfrac{\cos A}{(k - 4)(k + 1)} = \dfrac{\cos B}{k^2 + 2}$

$= \dfrac{\cos C}{(k + 4)(k - 1)}$

❖❖❖ ❖❖❖